$3°/\ (\overrightarrow{CB}\ ,\ \overrightarrow{AH}) \equiv (\overrightarrow{CB}\ ,\ \vec{i}) + (\vec{i}\ ,\ \overrightarrow{AH})\ (2\pi)$

$\equiv -\arg(z_B - z_C) + \arg(z_H - z_A)\ (2\pi) \equiv \arg(\dfrac{z_H - z_A}{z_B - z_C})\ (2\pi)$

$\equiv \arg(\dfrac{b+c}{b-c})\ (2\pi)$ Or $\dfrac{b+c}{b-c} = t$ d'où $(\overrightarrow{CB}\ ,\ \overrightarrow{AH}) \equiv \arg t\ (2\pi)$

b) t étant un nombre imaginaire pur donc $\arg t \equiv \dfrac{\pi}{2}\ (2\pi)$

ou $\arg t \equiv -\dfrac{\pi}{2}\ (2\pi)$ d'où $(\overrightarrow{CB}\ ,\ \overrightarrow{AH}) \equiv \dfrac{\pi}{2}\ (2\pi)$ ou

$(\overrightarrow{CB}\ ,\ \overrightarrow{AH}) \equiv -\dfrac{\pi}{2}\ (2\pi)$

Conclusion : $(CB) \perp (AH)$

Solution 7 :

$1°/\ |-1+i| = \sqrt{2}$; on a : $\cos\alpha = \dfrac{-\sqrt{2}}{2}$, $\sin\alpha = \dfrac{\sqrt{2}}{2}$

d'où $\alpha \equiv \dfrac{3\pi}{4}\ [2\pi]$ d'où $z = \sqrt{2}(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4})$.

$2°/\ z = \dfrac{1+i\,\mathrm{tg}\,\alpha}{1-i\,\mathrm{tg}\,\alpha} = \dfrac{1+i\dfrac{\sin\alpha}{\cos\alpha}}{1-i\dfrac{\sin\alpha}{\cos\alpha}} = \dfrac{\cos\alpha + i\sin\alpha}{\cos\alpha - i\sin\alpha}$

$z = \dfrac{[1,\alpha]}{[1,-\alpha]} = [1,2\alpha]$ d'où $z = 1\cdot(\cos(2\alpha) + i\sin(2\alpha))$.

$3°/$ Soit $z_0 = \dfrac{\sqrt{3}-i}{1+i} = \dfrac{[2,\dfrac{-\pi}{6}]}{[\sqrt{2},\dfrac{\pi}{4}]} = [\sqrt{2},-\dfrac{5\pi}{12}]$

d'où $z_0 = \sqrt{2}(\cos(-\dfrac{5\pi}{12}) + i\sin(-\dfrac{5\pi}{12}))$

or $z = {z_0}^n$ d'où $z = \sqrt{2}^n\left[\cos\dfrac{-5\pi n}{12} + i\sin\dfrac{-5\pi n}{12}\right]$

$4°/\ z = 1+i+(1-i)\,\mathrm{tg}\,\theta = 1+\mathrm{i}+\dfrac{(1-\mathrm{i})\sin\theta}{\cos\theta} = \dfrac{(1+i)\cos\theta + (1-i)\sin\theta}{\cos\theta}$

$z = \dfrac{\cos\theta + i\cos\theta + \sin\theta - i\sin\theta}{\cos\theta} = \dfrac{(\cos\theta - i\sin\theta) + (i\cos\theta + \sin\theta)}{\cos\theta}$